Turnstile Reflecto-Polarimeter Using the Principal Incidence Method: Determination of Permitivities Up to 1200°C and Industrial Applications

Alain Bretenoux, Cl. Marzat, and René Sardos

Abstract— **Having designed a free-space reflecto-polarimeter at 10 and 35 GHz using circular horns and turnstile analyzers, the authors proceed to use this device to measure the dielectric constants of various substances at temperatures up to 1200°C. Prismatic samples are used to eliminate the influence of the back face of the sample as well as that of interference phenomena. Some results are given, and also a practical application of principal- or Brewster-angle incidence to the heating of substances using microwaves is described.**

I. Introduction

The optical method using Brewster or principal incidence can be transferred to the microwave domain [1], [2], [3]. This method is of great interest for determining the dielectric constants of substances at high temperatures; however, some experimental difficulties are presented that reduce its precision. These difficulties, not including the inhomogeneity of the temperature, which we will not discuss here, are mostly due to multiple reflections, nonparallel beams, and analyzer defects.

It will be shown that it is possible to eliminate the multiple reflections, which makes the method more precise and simplifies the equations. Next, the experimental setup is described. In this setup, great care was taken at the emission end to ensure that at the sample level the waves were indeed plane waves. Great precision was maintained at the reception end by using well-adapted turnstile ellipsometer analyzers.

Last, a few results will be given, and it will be shown that the knowledge of ε and its behavior leads to a method that enables one to heat a substance by always making the energy penetrate it at Brewster incidence.

II. Theory

A. Determining ε' and ε'' Using the Brewster- or Principal-Angle Method

In the microwave range, the complex refractive index is replaced by ε' and ε'', and, therefore, the incident angle i_1 and the refracted angle i_2 are related by the following equation:

$$\sin^2(i_1) = (\varepsilon' - j\varepsilon'')\sin^2(i_2).$$

If one illuminates simultaneously with two waves, where one, $E_{\|}$, is parallel to the incident plane and the other one, $E_{\perp}$, is perpendicular to it, in phase, and of the same amplitude, then the resulting reflected wave will be elliptical since the ratio $r_{\|}/r_{\perp}$ is a complex number ($r_{\|}$ and $r_{\perp}$ are the reflection coefficients) that can be written as

$$r_{\|}/r_{\perp} = a = A\ \exp(j\Psi).$$

Measuring a for a given incident angle i_1 makes it possible to calculate ε' and ε''. At the Brewster or principal angle, we have $i_1 = i_p$ and $\Psi = -\pi/2$ [1], [2], [4]. In this case, the ellipticity of the wave is $\tan\beta = A$, and we have

$$\varepsilon' = 1 + \cos 4\beta \tan^2 i_p \sin^2 i_p$$

and

$$\varepsilon'' = \sin^2 i_p \tan^2 i_p \sin 4\beta.$$

In practice, since the sample is horizontal (see Fig. 1), the reflecto-ellipsometer consists of an emitter and a receiver, each rigidly attached to the sighting arms of a vertical goniometer.

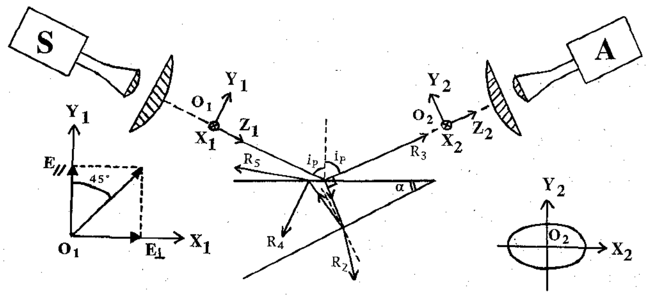

Fig. 1. Reflecto-ellipsometer—S: source; A: analyzer.

Thus the knowledge of i_p and of the ellipticity A gives ε' and ε'' very simply and without any approximations by using the preceding formulas.

These equations are valid in the case of a single refractive surface. However, in general, the sample consists of a plate with two parallel faces in which multiple reflections occur, and then the calculations become much more complicated.

B. Eliminating Errors Due to Multiple Reflections in the Plate

Starting with the Airy formulas [1], calculations show that one has to introduce two equivalent thicknesses (taking into account the phase jumps at reflection) to handle the multiple reflections. However, these calculations are useless if the slide thickness has not been measured with great precision, which is difficult in the case of very hot, or very cold, materials. In

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The authors are with the Laboratoire de Physique Experimentale et Micro-Ondes, Université Bordeaux, Talence, Cedex, France.
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the case of a very absorbent substance [4], one can use a plate thick enough to be able to neglect the multiple reflections.

It is easier and more rational to eliminate the influence of multiple reflections and of the plate's thickness by using only one single dielectric interface, which can be obtained by using a prismatic plate (see Fig. 1) with an angle $\alpha < \pi/2 - i_p$. Since the directions of beams R2, R4, and R5 are very different from the direction of beam R3 which enters the analyzers, R2, R4, and R5 are easy to eliminate.

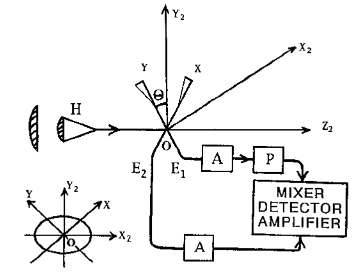

Fig. 2. Second type of analyzer—A: attenuator; P: phase shifter; H: circular horn.

III. Experimental Setup

A. Emitters

The first experiments done with horns with square or rectangular sections showed the existence of one very broad main lobe and of very strong sidelobes. These horns have been replaced by horns with a circular section whose sidelobes are weaker than 25 dB.

To improve the performance of the horns in the 10-GHz setup, a lens was placed in each horn in such a way that its focus coincided with the phase center of the horn. These lenses were designed especially to have the smallest possible VSWR, which was obtained by selecting an appropriate thickness. With such a lens the gain is improved by 3.2 dB as compared with the horn by itself, whereas the half-power lobe width of the emitted lobe is 11°.

In the 35-GHz setup, a second lens with a focal length of 25 cm was added to each horn. In this case the parameters of the setup are as follows: the half-power width of the main lobe is 3.5°, the sidelobes are weaker than 40 dB, and the maximum gain is improved by 40 dB.

Conversely, a study of the wavefront showed that the beam was parallel for a distance of about 15 cm. At the level of the sample, the beam has a width of 2.7 cm at 3 dB, and a width of 6.5 cm at 20 dB. The sidelobes are very weak and very well separated from the mainlobe, so that they are easy to eliminate.

B. Receivers and Analyzers

The wave reflected by the surface of the sample is elliptically polarized, and, at the point of principal incidence, one of the axes of the ellipse is in the incident plane whereas the other one—the big axis—is perpendicular to this plane.

The receiving horns are identical to the emitting horns; they are circular horns with lenses. Therefore, the elliptically polarized wave arrives in a circular waveguide.

The first element of the analyzer is a polarization duplexer. We chose to use a turnstile duplexer rather than a "fin-line" duplexer because the latter type has defects that can lead to errors of up to 1.8° [5]. Two types of turnstile junctions were used: the first, classical; the other of a more unusual type.

At 10 GHz, a nonclassical "lossy" turnstile junction was used [6], [7]. It has a broader transmission band and is easier to tune. The matrix used is the nonunitary S_1'' scattering matrix:

$$S_1'' = \frac{1}{4}\begin{bmatrix} 1 & -1 & 1 & -1 & 2\sqrt{2} & 0 \\ -1 & 1 & -1 & 1 & 0 & 2\sqrt{2} \\ 1 & -1 & 1 & -1 & -2\sqrt{2} & 0 \\ -1 & 1 & -1 & 1 & 0 & -2\sqrt{2} \\ 2\sqrt{2} & 0 & -2\sqrt{2} & 0 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 & -2\sqrt{2} & 0 & 0 \end{bmatrix}.$$

At 35 GHz, a classical turnstile junction was used which is tuned according to the usual S_1 matrix [8].

It is to be noted that whichever of these two turnstile junctions is used, lossy or classical, the wave entering the circular arm is decomposed in exactly the same way in arms 1 and 2, so that the calculations are the same for both frequencies.

1) First Type of Turnstile Analyzer Used (at 10 GHz and 35 GHz): In the case of Fig. 1, at Brewster or principal incidence the reflected wave is elliptically polarized, with one axis in the plane of incidence and the other perpendicular to this plane. To determine exactly when this angle is reached, one can either look for the energetic minimum of the vertically polarized part of the wave or determine the point at which the ellipticity is smallest by using an ellipsometer [4]. The latter method is more accurate.

The ellipsometer consists of a turnstile junction whose arms lie along the OX_2 and OY_2 axes of Fig. 1, and their directions are oriented as the axes of the ellipse at Brewster incidence.

Some difficulties appear for transparent and weakly absorbent substances because then one has to measure very small ellipticities, with $\tan\beta = b/a$ vanishingly small. Subsidiary measurements showed that this type of ellipsometer can be used to measure ellipticities as small as 2° with a precision of 0.5°.

Still, this classical setup is really useful only for absorbent substances. However, there is another more serious difficulty that is not immediately apparent. It is due to the fact that turnstile junctions are not perfect and do not totally separate the two orthogonal waves, as theory would have it.

To correct the shortcomings of this type of analyzer, so as to be able to use it in the case of small ε'', a second type of interferometric turnstile analyzer was designed by the authors.

2) Second Type of Interferometric Turnstile Analyzer (at 10 GHz and 35 GHz): In this case the turnstile junction is placed in such a way that its arms are at 45° of the incident plane. The axes of the reflected elliptical wave bisect the arms if and only if the wave comes in at principal incidence, Fig. 2.

If an elliptical wave is represented on the OX_2OY_2 reference system by the following vector with complex amplitudes:

$$\begin{pmatrix} a \\ -j\xi b \end{pmatrix}$$

(with $\xi = +1$ or $\xi = -1$ for a left- or right-handed ellipse, respectively) and this wave arrives in the turnstile junction whose arms are directed in the OX_2 and OY_2 directions, it splits into two waves whose complex amplitudes are proportional to (with θ being the angle between OY and OY_2) $E_1 = (a\sin\theta - j\xi b\cos\theta)$, which can be written $E_1 = Be^{-j\xi\Psi}$, and $E_2 = (a\cos\theta + j\xi b\sin\theta)$, which can be written $E_2 = Ae^{j\xi\varphi}$. If these two waves interfere in a square-law "mixer," they will take on a dephasing η with respect to each other. This dephasing can be modified by using a phase shifter. The total wave can now be written as

$$E = Be^{-j\xi\Psi} + Ae^{j\xi\varphi}e^{j\eta}.$$

The intensity seen by the receiver is

$$\begin{aligned} I = EE^* &= A^2 + B^2 + 2AB\cos[\xi(\Psi+\varphi)+\eta] \\ &= A^2\{(B^2/A^2) + \cos[\xi(\Psi+\varphi)+\eta](B/A) + 1\} \end{aligned}$$

provided A is not zero.

One must now find the conditions at which the intensity I vanishes, that is, $I = 0$. Let

$$\delta' = \cos^2[\xi(\Psi+\varphi)+\eta] - 1 = -\sin^2[\xi(\Psi+\varphi)+\eta],$$

therefore, $\delta' \le 0$. To obtain a real solution (nonimaginary), necessarily

$$\begin{aligned} \delta' = 0 &\Leftrightarrow \sin[\xi(\Psi+\varphi)+\eta] = 0 \\ &\Leftrightarrow \xi(\Psi+\varphi)+\eta = k\pi, \end{aligned}$$

hence $\eta = k\pi - \xi(\Psi+\varphi)$. There is one solution:

$$B/A = -\cos[\xi(\Psi+\varphi)+\eta] = \pm 1,$$

and since $\tan\beta = b/a$ with $0 \le \beta \le \pi/4$ and $a \ge 0$, and $b \ge 0$, it follows that $A \ge 0$ and $B \ge 0$, so that $A = B$, which leads to

$$\begin{aligned} a^2\cos^2\theta + b^2\sin^2\theta &= a^2\sin^2\theta + b^2\cos^2\theta \\ (a^2 - b^2)(\cos^2\theta - \sin^2\theta) &= 0. \end{aligned}$$

There are two possibilities:

1) $a = b$, but that means the wave has circular polarization; this is not the case here.
2) $a \neq b$; therefore, $\cos(2\theta) = 0$, and thus $\theta = \pi/4 + k\pi/2$.

This means that when $I = 0$, the wave is elliptical and its axes bisect the arms of the turnstile. The ellipicity and its right- or left-handedness are determined by measuring the dephasing η [9]. This ellipticity and the value of i_p make it possible to calculate the values of ε' and ε''.

Therefore, using a perfect duplexer whose arms are set at 45° of the vertical incident plane, it is easy to determine the angle at which the wave is reflected at the Brewster or principal angle (in the case studied here, one of the axes of the ellipse is in the vertical plane, and the other is horizontal). Indeed, all one needs to do is to find the point of extinction by varying the incident angle and the dephasing. In the case of an imperfect duplexer that does not completely separate the two orthogonal components of the wave, a fraction of each component perturbs the other one with a dephasing γ. The complex amplitude of the waves in each arm is then proportional to:

In arm 1: $E_1' = Be^{-j\xi\Psi} + kAe^{j\xi\Psi}e^{j\gamma}$

In arm 2: $E_2' = Ae^{j\xi\varphi} + kBe^{-j\xi\Psi}e^{j\gamma}$.

These waves then interfere in a mixer where they take on a dephasing η, which can be modified, just as in the first case. After some lengthy calculations, it can be shown that, to a second-order approximation, a zero minimum is obtained under the same conditions as for a perfect duplexer.

Additional measurements showed that it was possible to determine ellipticities of 100′ with an accuracy of 1′ at 10 GHz, and with an accuracy of 2′ at 35 GHz. Conversely, the 45° angle between the axes of the ellipse and the arms of the turnstile junction could be obtained with a precision of 2′ at 10 GHz and of 3′ at 35 GHz. These accuracies are determinant for the precision on i_p and on ε' and ε''; therefore, they will be indicated for each experiment.

Measurements done on well-known substances—water at 20°, bakelite, and alumina—confirmed the accuracy of the analyzer.

TABLE I
MEASUREMENTS ON POTASSIUM CHLORIDE

	ε'	$\Delta\varepsilon'$	ε''	$\Delta\varepsilon''$
Potassium chloride at 600°C and less	5.7	0.1	0.04	0.01
Lithium chloride at 500°C	14.7	0.5	32.0	0.8

IV. Results and Applications

A. Results

The setup having been tested at 20°C, the measurements were taken at increasing temperatures. Here, only results obtained at 35 GHz will be given since the oven used was too small to do measurements at 10 GHz. The oven could reach a maximum temperature of 1200°C and it was stable within 2°C. Two thermocouples, placed at a distance of 1 cm of each other, made it possible to control the temperature gradient and to adjust it to a zero value by manually regulating the surface heating.

In the case of potassium chloride, ε' and ε'' vary only slightly with temperature once the salt has crystallized (see Table I). When the sample is heated beyond the point of fusion (790°C), there is a sudden change. This phenomenon will be discussed in a later publication.

At room temperature, white alumina brick presents an abnormally high value for $\tan\delta$, namely, $\tan\delta = \varepsilon''/\varepsilon' = 0.21$ (see Table II). This is caused by absorption. The increase of ε'' between 500°C and 730°C is due to impurities and defects. White homogeneous and slightly porous sillimanite brick is a good dielectric material at 865°C. Measurements

TABLE II
MEASUREMENTS ON REFRACTORY MATERIALS CONTAINING ALUMINA

Sample	t, °C	ε'	$\Delta\varepsilon'$	ε''	$\Delta\varepsilon''$
Industrial	20	2.38	0.05	0.50	0.02
Alumina	500	3.17	0.05	0.15	0.01
Brick	730	0.17	0.05	0.30	0.02
Sillimanite	20	6.12	0.08	0.21	0.02
Brick	865	6.12	0.08	0.06	0.01

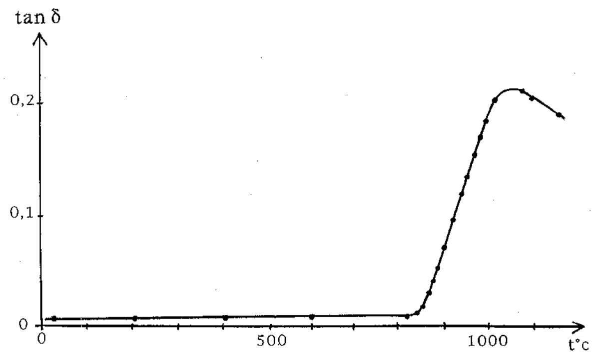

Fig. 3. $t_g\delta$ versus t for pure fritted alumina.

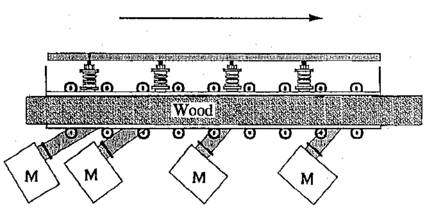

Fig. 4. Brewster incidence applicators—M: magnetron.

done on less homogeneous bricks show large losses at room temperature, which increase as the temperature increases.

The graph in Fig. 3 shows the variations of $\tan\delta$ as a function of temperature for a sample of pure fritted alumina: a rapid increase is observed beyond 800°C.

B. Industrial Applications

For reasons of energetic yield, microwave energy should penetrate a substance at principal incidence with the appropriate polarization. Having a knowledge of the principal angle at various temperatures is therefore essential.

The industrial applications are numerous. One example is the drying [11] and bonding [12] of wood where microwaves are used for thermic transfer. Such an apparatus is illustrated in Fig. 4.

It is seen that the incident angle of the waveguide applicators changes during the drying process (it diminishes) to account for the variations of the dielectric constant as the residual humidity decreases. The microwave generators are placed further and further apart, at a distance just sufficient to maintain a thermal gradient large enough for the water to transfer from the core to the highly ventilated surface.

V. CONCLUSION

The measuring method described here, together with the apparatus designed by the authors, allow the determination of ε' and ε'' at high temperatures. The method should be easy to automate since it is a null method involving only two parameters, the incident angle and the dephasing.

A knowledge of the value of the Brewster angle at various temperatures provides an important characterization of a substance. The applications of this knowledge are manifold. One of these is the possibility of making the microwave energy penetrate a material at the most effective angle to produce the best yield.

REFERENCES

[1] M. Born and E. Wolf, *Principles of Optics.* New York: Pergamon, 1959.
[2] G. Bruhat and A. Kastler, *Optique.* Paris: Masson, 1965.
[3] IRMM 87 International Conference on Infrared and Millimeter-Waves.
[4] A. Bretenoux, "Réalisation d'un réflecto-polarimètre à 35 GHz pour la mesure à l'air Libre des constantes diélectriques de divers matériaux portés à hautes températures," Third Cycle Thesis, Bordeaux, 1967.
[5] R. J. Vernon and T. A. Durschner, "An improved interferometric polarization analyzer for measuring the microwave magneto-Kerr effect in semiconductors," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-19, p. 287, 1971.
[6] E. Rivier and R. Sardos, *La matrice S du numérique à l'Optique.* Paris, New York: Masson, 1982.
[7] R. Sardos, J-F. Escarmant, Y. Ramdani, A. Balana, and W. J. Ellison, "Using a 16 GHz interferential reflecto-ellipso-polarimeter to study magnetic composite media," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-38, pp. 313–318, 1990.
[8] J. L. Altman, *Microwave Circuits.* London: Van Nostrand, 1964.
[9] R. Sardos, "Réalisation d'un polarimètre-ellipsomètre et d'un réflecto-polarimètre, pour la bande des 10 GHz," Thesis, Bordeaux, 1965.
[10] C. Marzat, "Microwave applicator under Brewster incidence," French Patent F.8904572.
[11] C. Marzat and P. A. Bernard, *Proc. 3rd Conf. Wood Sciences and Industry.* Arbora publisher, Book I, 1991, p. 155.
[12] C. Marzat, P. A. Bernard, and R. Decourt, "Microwave bonding in the wood industry," in *Proc. Conf. Microwaves and High Frequencies.* CFE Publisher, Nice, vol. 2, 1991, p. 113.

Alain Bretenoux was born in 1944 near Bordeaux, France. He obtained a Third Cycle Thesis in 1967.

Presently, he is "Maitre de Conférences" and works with the "Laboratoire MASTER" at ENSCPB of the Bordeaux University.

C. Marzat, Professor at the University of Bordeaux, ENSERB engineer, Senior Lecturer at the Department of Electronics and Electronic Engineering at the University Institute of Technology, is the Director of the Laboratory of Applied Physics and Microwaves, in charge of industrial microwave applications.

René Sardos was born in 1927 in the Bordeaux area of France. He obtained the doctorat d'etat (science and physics) in 1965 from the University of Bordeaux.

Since 1967 he has been a Professor at the University of Bordeaux and the head of the microwave interferometry and polarimetry team. His present research interests include the magnetic properties of composite materials, magnetic anisotropy, and magnetoreflection in ferrites and semiconductors.